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HEAT AND MASS TRANSFER AND PHYSICAL GASDYNAMICS

Excitation of Acoustic Vibrations during Volume Condensation in a Steam–Gas Mixture

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Abstract—A theoretical investigation is performed of the acoustic instability under conditions of volume condensation in a steam–gas mixture. The steam–gas mixture is assumed to be stationary and consist of a gas and moist steam. It is assumed that gas and steam are ideal gases and that the condensation process is nonequilibrium. Condensate droplets are assumed to be spherical and monodisperse, and their volume concentration is assumed to be minor compared to unity. It is demonstrated that acoustic vibrations are excited during volume condensation of steam. Formulas are derived for the calculation of the increment and frequency of vibrations.

INTRODUCTION

It is well known that acoustic vibrations may have a considerable effect on the processes of heat and mass transfer and on the mechanical strength of structures [1, 2]. Therefore, a study of the conditions of excitation of acoustic vibrations in steam–gas mixtures and analysis of their characteristics is of doubtless interest. This paper deals with the investigation of excitation of acoustic vibrations during condensation of the products of coal combustion in the gasdynamic flow train of a pulsed MHD generator [3].

When pulverized coal is used as fuel, partially condensed slag vapors consisting mainly of silicon oxide will be present in the MHD-generator channel [4]. Because of a considerable temperature gradient (~1000 K/m) along the channel, associated with the operating conditions of the channel, the condensation of slag vapors is volumetric. The process of volume concentration is known [5] to consist of the processes of nucleation and growth of condensed particles. It is usually assumed [5] that all nuclei are formed at the beginning of condensation, and the desupersaturation occurs at the beginning of the process of growth of condensed particles.

Previously [6, 7], I investigated the acoustic instability during phase transitions in steam–gas mixtures. In these studies, I used the energy method [8] to investigate the acoustic stability; this method makes it possible to reveal sources of vibrations in every concrete case and, in so doing, enables one to attain only the sufficient conditions for the system stability. In this study, which is a continuation and refinement of [6, 7], it is assumed that the steam–gas mixture (products of pulverized coal combustion in oxygen and seed vapors) is homogeneous, and solutions for small perturbations that have been derived by the method of separation of variables are investigated. This approach to the solution of the problem on excitation of vibrations enables one to derive formulas for the increment of vibrations and their frequency depending on the characteristics of the two-phase mixture and of the MHD-generator channel (rate of nucleation, size of condensed particles, length of the MHD-generator channel) and is a widely used method of studying vibrations [8].

BASIC EQUATIONS

The basic equations describing the excitation of acoustic vibrations during condensation of slag in the channel of an MHD generator were written with the following basic assumptions. The steam–gas mixture was taken to be an ideal gas, and the concentration of vapor (silicon oxide) in this mixture was taken to be insignificant, and its effect on the thermal properties of the mixture (which were assumed to be independent of temperature) was ignored. The condensate particles were taken to be spherical, monodisperse, and stationary relative to the steam–gas mixture.

In view of these assumptions for a one-dimensional formulation of the problem, one can write the following equations of gas dynamics:

$$\frac{\partial\rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) = -W,$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = -\frac{1}{\rho}\frac{\partial p}{\partial x},$$

$$\rho c_p\left(\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x}\right) = \frac{\partial p}{\partial t} + LW,$$

where W is the rate of condensation per unit volume, L is the heat of condensation, and u is the velocity of the steam–gas mixture.

The dependence of the mixture density on the thermodynamic parameters will be written as

$$\rho = \rho(p, T, \beta). \tag{1}$$

Here, β is the dimensionless mass concentration of gas in the steam–gas mixture,

$$\beta = \rho_2/(\rho_1 + \rho_2).$$

The subscript 1 indicates steam, and the subscript 2 indicates gas.

The experience of development of liquid-propellant aircraft and rocket engines shows [8] that high-frequency acoustic vibrations are most dangerous and hard to eliminate. Therefore, I investigate the excitation of high-frequency acoustic vibrations in the channel of an MHD generator during volume condensation of silicon oxide.

We linearize the equations of gas dynamics to derive the approximate equations

$$\frac{\partial \rho'}{\partial t} + \rho\frac{\partial u'}{\partial x} + u'\frac{\partial \rho}{\partial x} = -W', \tag{2}$$

$$\frac{\partial u'}{\partial t} = -\frac{1}{\rho}\frac{\partial p'}{\partial x}, \tag{3}$$

$$\rho c_p\left(\frac{\partial T'}{\partial t} + u'\frac{\partial T}{\partial x}\right) = \frac{\partial p'}{\partial t} + LW'. \tag{4}$$

Here and below, the perturbations are indicated by primes.

Note that approximate equations (2)–(4) may be used to study high-frequency acoustic vibrations in a moving medium provided the inequality $\mathrm{Sh} = \omega l/u \gg 1$ is valid (Sh is the Strouhal number for acoustic vibrations, ω is the circular frequency, and l is the characteristic dimension).

Further, we linearize Eq. (1) to derive

$$\frac{\rho'}{\rho} = \frac{p'}{\rho c_T^2} - \frac{T'}{T} + \frac{1}{\rho}\left(\frac{\partial \rho}{\partial \beta}\right)_{p,\,T}\beta', \tag{5}$$

where $c_T = [(\partial p/\partial \rho)_{\beta,\,T}]^{1/2}$ is the isothermal velocity of sound.

In order to calculate $(\partial\rho/\partial\beta)_{p,\,T}$, we will write the equation of state for gas mixture,

$$p = \left(\frac{1-\beta}{\mu_1} + \frac{\beta}{\mu_2}\right)RT.$$

Here, μ is the molecular weight of a component of the steam–gas mixture.

We differentiate the latter expression at T = const and p = const to derive

$$\left(\frac{\partial \rho}{\partial \beta}\right)_{p,\,T} = \frac{\rho(\mu_2 - \mu_1)}{(1-\beta)\mu_2 + \beta\mu_1}. \tag{6}$$

The substitution of expression (5) into Eq. (2) in view of Eqs. (3) and (6) gives

$$\frac{1}{\gamma}\frac{\partial}{\partial t}\left(\frac{p'}{p}\right) + \frac{\partial u'}{\partial x} = \left[\frac{2\beta(\mu_2-\mu_1)-\mu_2}{(1-\beta)\mu_2+\beta\mu_1} + \frac{L}{c_pT}\right]\frac{W'}{\rho},$$

where γ is the adiabatic exponent of the steam–gas mixture.

We differentiate the latter equation with respect to t and Eq. (3) with respect to x and eliminate u' to derive

$$\begin{aligned}\frac{1}{\rho}\frac{\partial^2 p'}{\partial x^2} &= \frac{1}{\gamma}\frac{\partial^2}{\partial t^2}\left(\frac{p'}{p}\right)\\ &-\frac{1}{\rho}\left[\frac{2\beta(\mu_2-\mu_1)-\mu_2}{(1-\beta)\mu_2+\beta\mu_1} + \frac{L}{c_pT}\right]\frac{\partial W'}{\partial t}.\end{aligned} \tag{7}$$

In order to solve this equation, one needs to derive the expression for W' which, in the general case, depends on perturbations of nucleation rates and on further growth. According to the classical nucleation theory [5], the number of nuclei formed per unit volume per unit time is defined by the expression

$$J = \frac{1}{\rho_k}\left(\frac{p_1}{kT}\right)^2\left(\frac{2\sigma\mu_1}{\pi N_\mathrm{A}}\right)^{1/2}\exp\left[-\frac{4\pi(r^*)^2\sigma}{3kT}\right], \tag{8}$$

in which k is the Boltzmann constant, N_A is Avogadro's number, p_1 is the vapor pressure, ρ_k is the liquid density, σ is the surface tension, and r^* is the radius of the critical nucleus.

The critical nucleus size is calculated by Thomson's formula [5],

$$r^* = \frac{2\sigma\mu_1}{\rho_k RT\ln(p_1/p_{1\infty})}, \tag{9}$$

where $p_{1\infty}$ is the saturated vapor pressure.

The value of W by the moment of time t may be determined as follows, assuming that the growing droplets in every channel section are monodisperse [9]:

$$W = \int_{t_1}^{t} J(t')\frac{d}{dt}m(t, t')dt'.$$

Here, $J(t')$ is the number of nuclei formed per unit volume per unit time at the moment of time t', and m is the mass of a droplet that grew from a nucleus which emerged at the moment of time t', $m = 4/3\pi\rho_k r^3$.

The bulk of nuclei are formed at the moment the steam–gas mixture passes through the supersaturation maximum, which is associated with the sharp tempera-

ture dependence of J [9]. Note further that the time of nucleation from the supersaturated steam–gas mixture is much less than the time of condensation growth of liquid droplets [9]. Therefore, the latter expression may be approximately written as

$$W = \frac{4}{3}\pi\rho_k r^3(t)J(t'). \tag{10}$$

In order to derive the formula for W', the correlation between the perturbations of temperature T' and pressure p' must be obtained from the latter expression. We will assume that T' and p' are related by the known thermodynamic relation (quasi-adiabatic approximation [8]),

$$\frac{T'}{T} = \frac{\gamma - 1}{\gamma}\frac{p'}{p}. \tag{11}$$

The value of W', which depends on the perturbations of pressure and temperature of the steam–gas mixture in the case of acoustic vibrations, is largely defined by the perturbation of $J(t')$. This is due, as was already mentioned above, to the very sharp dependence of $J(t')$ on the temperature of the steam–gas mixture. Then, we linearize Eq. (10) in view of (8), (9), and (11) to derive

$$W' = W\frac{\sigma(r^*)^2}{kT}\frac{\gamma-1}{\gamma}\frac{p'}{p}.$$

The substitution of this expression into Eq. (7) gives

$$\frac{\partial^2}{\partial x^2}\left(\frac{p'}{p}\right) = \frac{1}{c_s^2}\frac{\partial^2}{\partial t^2}\left(\frac{p'}{p}\right) - \alpha\frac{\partial}{\partial t}\left(\frac{p'}{p}\right), \tag{12}$$

where $\alpha = \frac{W}{\rho c_s^2}\left[\frac{2\beta(\mu_2-\mu_1)-\mu_2}{(1-\beta)\mu_2+\beta\mu_1} + \frac{L}{c_pT}\right]\frac{\sigma(r^*)^2}{kT}\frac{\gamma-1}{\gamma}$, and c_s is the adiabatic velocity of sound.

In order to determine the frequency of acoustic vibrations upon their excitation, one must preassign the boundary conditions to Eq. (12). I have made the simplest assumption that, from the acoustic standpoint, an MHD channel with a diffuser may be regarded as a pipe open at one end. This assumption may be taken to be valid because the area of the combustor nozzle is much less than the area of the diffuser exit section. With these assumptions, the boundary conditions will have the form

$$\partial p'/\partial x = 0 \text{ at } x = 0,$$

$$p' = 0 \text{ at } x = l.$$

Here, l is the length of the MHD channel with a diffuser.

We solve Eq. (12) by the method of separation of variables in view of the boundary conditions to derive

$$\frac{p'}{p} = e^{\delta t}\sum_{k=1}^{\infty} B_k \cos\left[\left(\frac{2k-1}{2}\right)\frac{\pi x}{l}\right] \times \cos\left(\sqrt{\omega_k^2-\delta^2}\,t + \varphi_k\right). \tag{13}$$

Here, $\delta = \alpha c_s^2/2$, and $k = 1, 2, 3, \dots$.

The values of B_k and φ_k are determined from the boundary conditions. The eigenfrequency of the MHD channel with a diffuser, determined from the boundary conditions, will be

$$\omega_k = \frac{2k-1}{2}\frac{\pi c_s}{l}. \tag{14}$$

DISCUSSION OF THE RESULTS

One can see in expression (13) that, at $\delta > 0$, the volume condensation of slag will cause the excitation of acoustic vibrations with the increment

$$\theta = \frac{2\pi\delta}{\sqrt{\omega_k^2-\delta^2}}.$$

As follows from Eq. (13), the excitation of acoustic vibrations on the channel of an MHD generator, caused by the volume condensation of slag, results in a decrease in the vibration frequency of the MHD channel with a diffuser compared with their eigenfrequency. For example, in the case of volume condensation of the slag of Grade G Kuznetskii coal, which shows promise for use in MHD generators [4], the value of δ is $\delta \sim 30\ \text{s}^{-1}$ with the fundamental tone $\omega_1 \approx 2000\ \text{s}^{-1}$. Note that the obtained solution of the problem on excitation of acoustic vibrations during condensation of slag in the channel of an MHD generator is limited by subsonic and low supersonic velocities. This is associated with the fact that the assumption that the steam–gas mixture is stationary ($\text{Sh} \gg 1$) is valid, as is seen from Eq. (14), for subsonic and low supersonic velocities of flow.

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